

Exercise 32

Find the derivative of the function.

$$F(t) = \frac{t^2}{\sqrt{t^3 + 1}}$$

Solution

Take the derivative using the quotient rule and the chain rule.

$$\begin{aligned} F'(t) &= \frac{dF}{dt} = \frac{d}{dt} \left(\frac{t^2}{\sqrt{t^3 + 1}} \right) \\ &= \frac{\left[\frac{d}{dt}(t^2) \right] \sqrt{t^3 + 1} - \left[\frac{d}{dt}(\sqrt{t^3 + 1}) \right] (t^2)}{t^3 + 1} \\ &= \frac{(2t)\sqrt{t^3 + 1} - \left[\frac{1}{2}(t^3 + 1)^{-1/2} \cdot \frac{d}{dt}(t^3 + 1) \right] (t^2)}{t^3 + 1} \\ &= \frac{2t\sqrt{t^3 + 1} - \left[\frac{1}{2}(t^3 + 1)^{-1/2} \cdot (3t^2) \right] (t^2)}{t^3 + 1} \\ &= \frac{2t\sqrt{t^3 + 1} - \frac{3t^4}{2\sqrt{t^3 + 1}}}{t^3 + 1} \\ &= \frac{\frac{4t(t^3 + 1)}{2\sqrt{t^3 + 1}} - \frac{3t^4}{2\sqrt{t^3 + 1}}}{t^3 + 1} \\ &= \frac{\frac{t^4 + 4t}{2\sqrt{t^3 + 1}}}{t^3 + 1} \\ &= \frac{t(t^3 + 4)}{2(t^3 + 1)^{3/2}} \end{aligned}$$